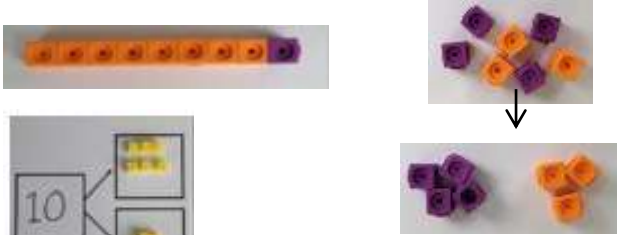
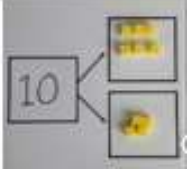


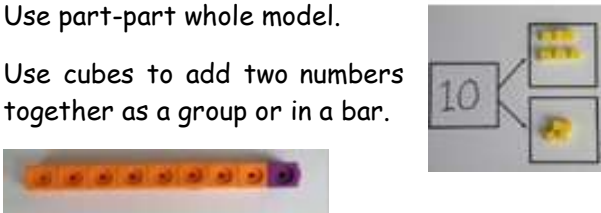

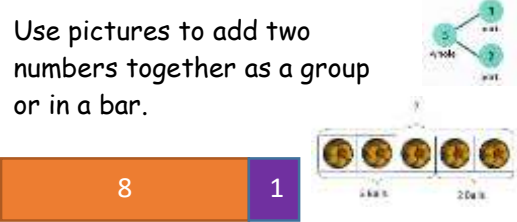
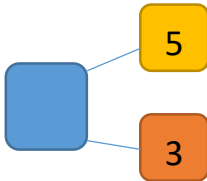
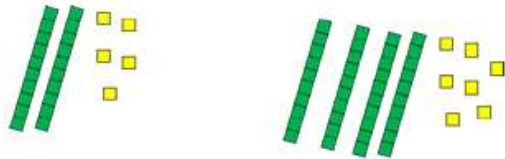




Addition	Concrete	Pictorial	Abstract
<p>Foundation</p>	<p>Use part-part whole model.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p>   <p>Simple word problems using their fingers.</p>  <p>$5 + 1 = 6$</p>	<p>Make a record in pictures, words or symbols of addition activities already carried out.</p> <p>Use pictures to add two numbers together as a group or in a bar.</p>  <p>Initially use a number track to count on for addition, counting on from the largest number:</p> <p>$8 + 7 = 15$ 'Put your finger on number eight and count on seven.'</p>	<p>Children will engage in a wide variety of songs, games and activities.</p> <p>They will begin to relate addition to combining two groups of objects, first by counting all of them and then from counting on from the largest number.</p> <p>Using quantities and objects children add two single digit numbers.</p> <p>Children may be introduced to written 'number sentence' e.g. $4 + 3 = 7$</p> <p>Construct number sentences to go with practical activities.</p>
<p>Year 1</p>	<p>Use part-part whole model.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p>  <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p> 	<p>Use pictures to add two numbers together as a group or in a bar.</p>  <p>Initially use a number track to count on for addition, counting on from the largest number:</p> <p>$8 + 7 = 15$ 'Put your finger on number eight and count on seven.'</p>	<p>Children will continue to practice counting on from any number e.g. 'Put five in your head and count on four.'</p> <p>Using the part-part whole diagram to move into the abstract</p> 



Year 2



Model using dienes, place value counters and Numicon




Regrouping to make 10.
 $6 + 5 = 11$

Start with the bigger number and use the smaller number to make 10.



Counting on in ones and tens using an empty number line, within 100...

$$28 + 5 = 33$$



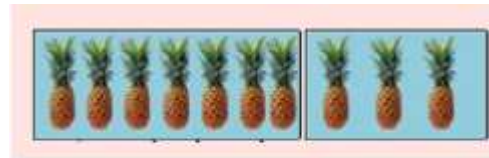
$$28 \quad 29 \quad 30 \quad 31 \quad 32 \quad 33$$

$$28 + 30 = 58$$



$$28 \quad 38 \quad 48 \quad 58$$

Also using Bar Model



$$7 + 3 = 10$$

$$25 + 47$$

$$\begin{array}{r} 20 + 5 \\ 40 + 7 \end{array}$$

$$20 + 40 = 60$$

$$5 + 7 = 12$$

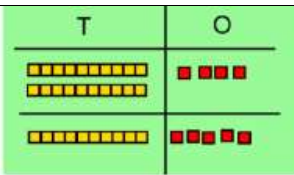
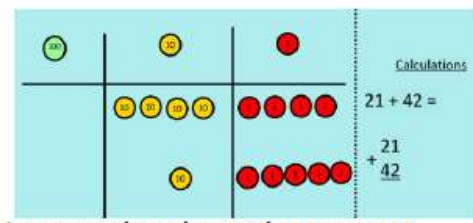
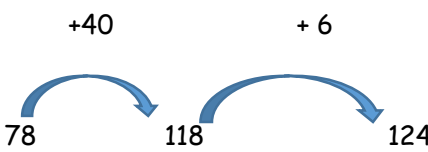
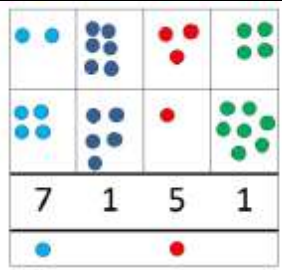
$$60 + 12 = 72$$

23	25
?	

$$23 + 25 = 48$$

Number line approach may also be modelled to children as another approach to addition.

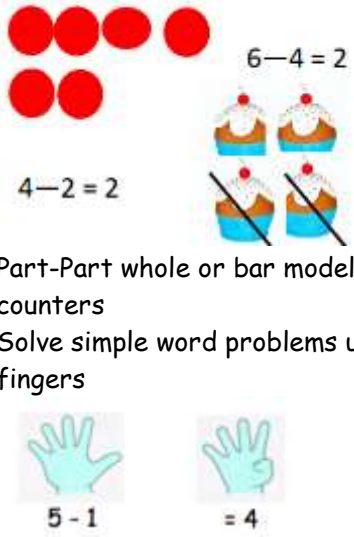



<p>Year 3</p>	<p>T O</p>  <p>Model using Dienes or num- micon</p> <p>Add together the ones first, then the tens.</p> <table border="1" data-bbox="436 486 817 710"> <thead> <tr> <th></th> <th>Tens</th> <th>Units</th> </tr> </thead> <tbody> <tr> <td>45</td> <td></td> <td></td> </tr> <tr> <td>34</td> <td></td> <td></td> </tr> <tr> <td></td> <td>7</td> <td>9</td> </tr> </tbody> </table>  <p>Move to using place value counters</p>		Tens	Units	45			34				7	9	<p>Further develop the use of the empty number line with calculations that bridge 100:</p> $78 + 46 = 124$  <p>Introduce column method</p> $63 + 32 = 95$ $\begin{array}{r} 60 + 3 \\ + 30 + 2 \\ \hline 90 + 5 = 95 \end{array}$	$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ <p>Add the ones first, then the tens, then the hundreds.</p>
	Tens	Units													
45															
34															
	7	9													
<p>Year 4</p>	<p>Continue to use dienes or Place Value counters for adding, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.</p> <table border="1" data-bbox="313 1101 862 1340"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </tbody> </table>	Hundreds	Tens	Ones							 <p>Draw representations using pv grid.</p>	<p>Further develop the formal written method of addition, with three-digit numbers. Revisit the expanded method first, if necessary.</p> $176 + 147 = 323$ $\begin{array}{r} 176 \\ + 147 \\ \hline 13 \text{ (6 + 7)} \\ 110 \text{ (70 + 40)} \\ \hline 200 \text{ (100 + 100)} \\ \hline 323 \end{array}$ <p>This will lead into the formal written method.</p> $1845 + 526 = 2371$ $\begin{array}{r} 1845 \\ + 526 \\ \hline 2371 \end{array}$			
Hundreds	Tens	Ones													



Year 5	Use Year 4 method if appropriate	Use Year 4 method if appropriate	<p>Continue to teach the use of empty number lines with larger numbers (and decimals), as appropriate.</p> <p>Continue to develop the formal written method for addition with larger numbers (and decimal numbers) and with the addition of three or more numbers.</p> <p>$£154.75 + £233.82 = £388.57$</p> $\begin{array}{r} 154.75 \\ + 233.82 \\ \hline 388.57 \end{array}$
Year 6	Use Year 4 method if appropriate	Use Year 4 method if appropriate	<p>Our aim is that by the end of Y6, children use mental methods (with jottings) when appropriate, but for calculations that they cannot do in their heads, they use an efficient formal written method accurately and with confidence.</p>

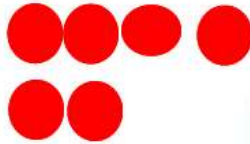


Subtraction	Concrete	Pictorial	Abstract
<p>Foundation</p>	<p>Use physical objects, counters ; cubes etc to show how objects can be taken away.</p>  <p>Part-Part whole or bar modelling used with counters Solve simple word problems using their fingers</p>	<p>Children draw representations of the objects. Including part-part whole or bar model.</p> 	<p>Children will engage in a wide variety of songs, games and activities Using quantities and objects children subtract two single digit numbers.</p> <p>Children may be introduced to written 'number sentence' e.g. $7 - 3 = 4$</p>



Year 1

Use physical objects, counters, cubes etc to show how objects can be taken away.



$$6 - 4 = 2$$

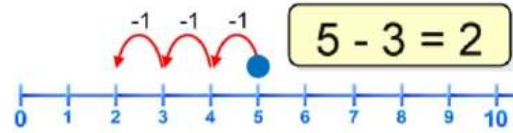
$$4 - 2 = 2$$



Part-Part whole or bar modelling used with counters

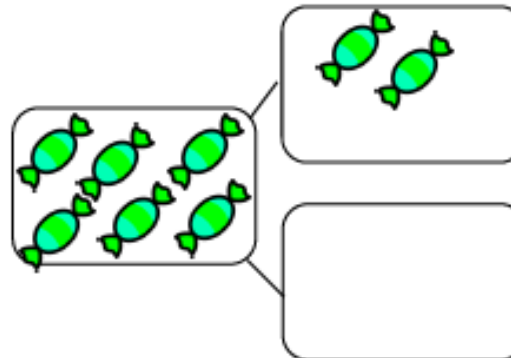


Use cubes to build towers or make bars to find the difference



Count back in ones using a number line.

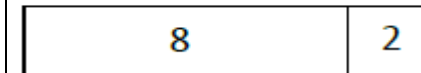
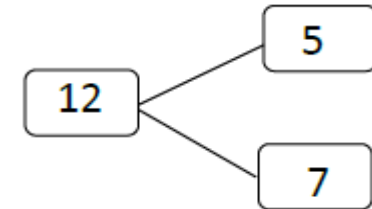
Also using number line to count on to find the difference



Use pictorial representations to show the part.

Putting number in head and counting back.

Move to using numbers within the part whole model.



$$10 = 8 + 2$$

$$10 = 2 + 8$$


$$10 - 2 = 8$$

$$10 - 8 = 2$$



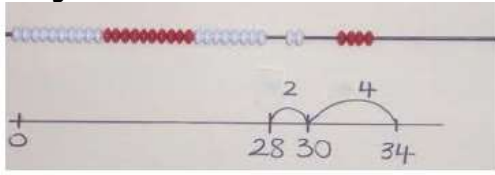
Year 2

$34 - 13 = 21$



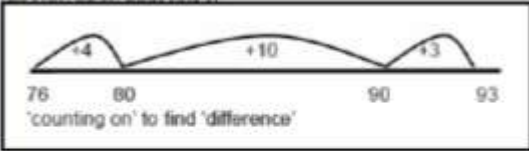
Use Dienes to show how to partition the number when subtracting without regrouping.

Finding difference




$34 - 28$

Use a bead bar or bead strings to model counting to next ten and the rest.



Use a number line to count on to next ten and then the rest.


Children draw representations of Dienes and cross off.



$43 - 21 = 22$


Counting back using an empty number line within 100, in ones...

$34 - 6 = 28$



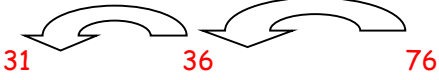
And in tens...

$58 - 30 = 28$

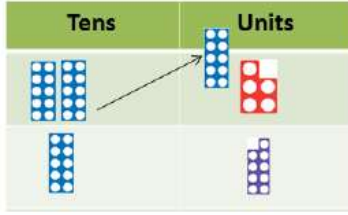


If children are confident, use efficient jumps:

$76 - 45 = 31$

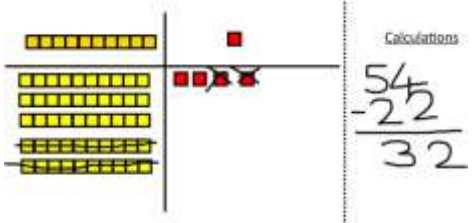


Year 3



Begin with dienes or Numicon. Move to PV counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.

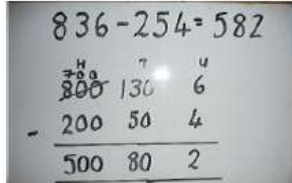
Children to continue to use number line approaches leading to strategy below.



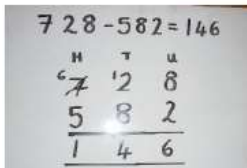
Calculations

$$\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$$

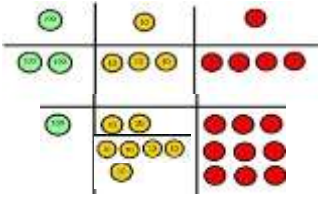

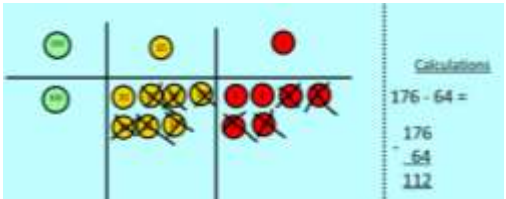
Begin by partitioning into PV columns



Then move to a formal method



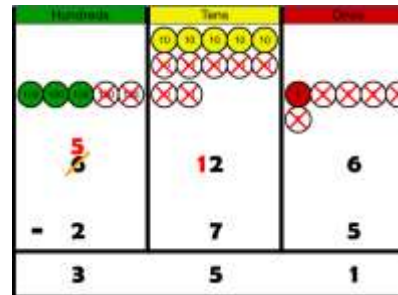


<p>Year 4</p>	<p style="text-align: center;">234 - 179</p>  <p>Model process of exchange using Numicon, base ten and then move to PV counters.</p>	<p>Draw the Base 10 or place value counters alongside the written calculation to help to show working.</p>   <p style="text-align: right;">Calculations 176 - 64 = 176 - 64 112</p>	<p>Continue to develop the formal written method for subtraction with three and four digit numbers, returning to an expanded method and using base ten materials, if necessary.</p> <p>Move onto larger numbers</p> $3625 - 1219 = 2406$ $\begin{array}{r} 115 \\ 3625 \\ - 1219 \\ \hline 2406 \end{array}$
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Year 5-6

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.



Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.



exchange/regrouping.

When confident, children can find their own way to record the



Continue to teach the use of empty number lines with larger numbers (and decimals), as appropriate.
Continue to develop the formal written method for addition with larger numbers (and decimal numbers) and with the addition of three or more numbers.

$$£154.75 + £233.82 = £388.57$$


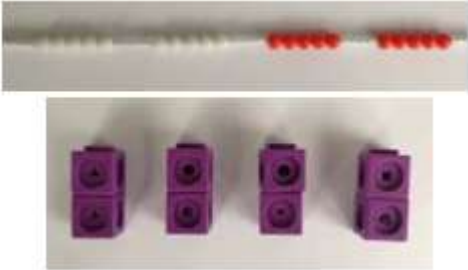
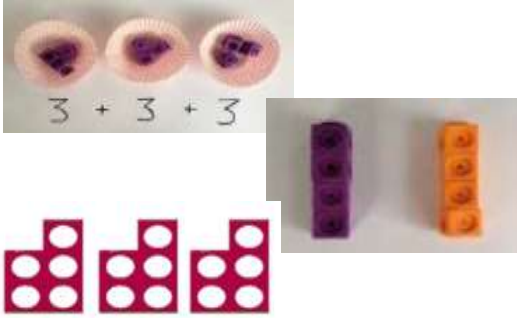
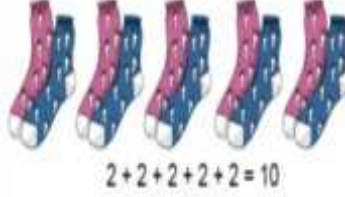

$$\begin{array}{r} 154.75 \\ + 233.82 \\ \hline 388.57 \\ 1 \end{array}$$

Ensure that the decimal points line up.




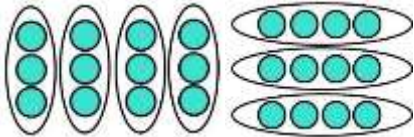
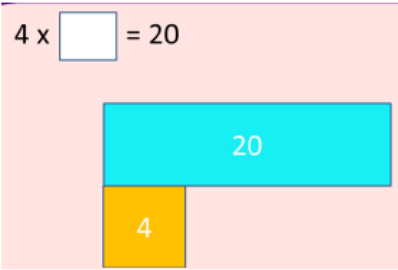



Multiplication	Concrete	Pictorial	Abstract
<p>Foundation</p>	<p>Children to use counters and through song, begin to count in a given multiple.</p>  <p>Part -part whole used with counters Begin to use resources to count in repeated groups of the same size: count in twos; fives; tens</p>	<p>Children draw visual representations of maths problems involving repeated addition and doubling.</p>	<p>Children shown multiplication number sentence alongside visual representation.</p> <p>Children explore different objects to make doubles- dice, spots on ladybirds. Children shown abstract 'number sentence' alongside visual representation.</p>  <p>Children are able to chant in twos, fives and tens</p>



<p>Year 1</p>	<p>In practical activities and through discussion they will begin to solve problems involving doubling.</p> <p>Three apples for me and three apples for you. How many apples altogether?</p>  <p>Children arrange counters for them to use to count in multiples.</p> 	 <p>Use different objects to add equal groups</p>	<p>Write addition sentences to describe objects and pictures.</p>  <p>Use pictorial including number lines to solve problems</p> <p>I have 2 toys in a box. How many toys would I have if I had 5 boxes altogether?</p> 
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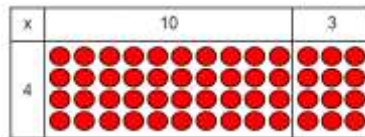


<p>Year 2</p>	<p>Create arrays using counters and cubes and Numicon.</p>   <p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</p> 	<p>Use representations of arrays to show different calculations and explore commutatively.</p>  <p>4×3 or 3×4</p> <p>Using Bar model approach to show missing number problems.</p> 	<p>$12 = 3 \times 4$</p> <p>$12 = 4 \times 3$</p> <p>Use an array to write multiplication sentences as well as a number line to reinforce repeated addition</p>  <p>$5 + 5 + 5 = 15$</p> <p>$3 + 3 + 3 + 3 + 3 = 15$</p> <p>$5 \times 3 = 15$</p> <p>$3 \times 5 = 15$</p>
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Year 3

Show the link with arrays to first introduce the grid method.

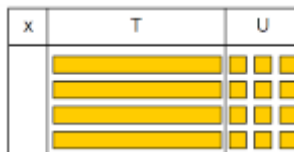


4 rows of 10

4 rows of 3

Move on to using Base 10 to move towards a more compact method.

4 rows of 13



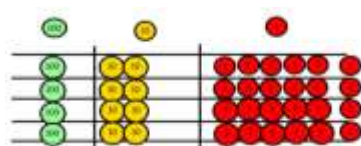
Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.



Calculations
4 x 126

Fill
each
row

with 126.



Calculations
4 x 126

Children can represent their work with place value counters. E.g. Counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.

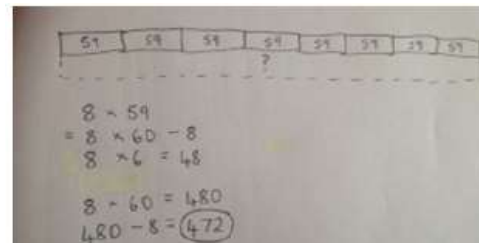


The grid method may be used to show how this relates to a formal written method.

x	300	20	7
4	1200	80	28

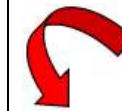


Bar Modelling to support children with problem solving.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

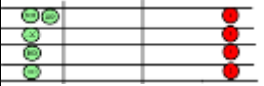

$$\begin{array}{r}
 327 \\
 \times 4 \\
 \hline
 28 \\
 80 \\
 1200 \\
 \hline
 1308
 \end{array}$$



	3	2	7
x			4
	1	3	0
		1	2
			8

This may lead to a compact method.

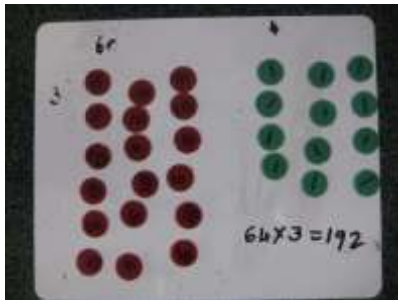


	<p>Add up each column, starting with the ones making any exchanges needed.</p>  <p>Then you have your answer.</p> 		
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Year 4-6

Children can continue to be supported by place value counters at the stage of multiplication



It is

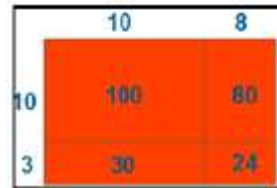
important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

The grid method may be used to show how this relates to a formal written method.

x	300	20	7
4	1200	80	28



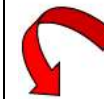
Continue to use bar modelling to support understanding.



Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.

Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

$$\begin{array}{r} 327 \\ \times 4 \\ \hline 28 \\ 80 \\ 1200 \\ \hline 1308 \end{array}$$



$$\begin{array}{r} 327 \\ \times 4 \\ \hline 1308 \\ \hline \end{array}$$

This may lead to a compact method.

Years 5 and 6 to develop compact formal method


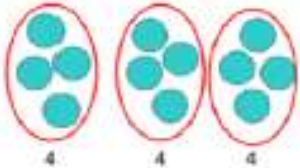
	1	8	
x	1	3	
	5	4	
	2		
	1	8	0
	2	3	4

18 x 3 on the first row
(8 x 3 = 24, carrying the 2 for 20, then 1 x 3)

18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first

$$\begin{array}{r} 1234 \\ \times 16 \\ \hline 7404 \quad (1234 \times 6) \\ 12340 \quad (1234 \times 10) \\ \hline 19744 \end{array}$$



Division	Concrete	Pictorial	Abstract
Year 1/EYFS	<p>Children will share objects into equal groups and through discussion they will begin to solve problems involving halving and sharing.</p>  <p>I have 10 cubes, can you share them equally into 2 groups? Children use counters with part-part whole model.</p>	<p>Children to use pictures to support their sharing of quantities.</p>  <p>12 shared between 3 is 4</p> <p>Children find $\frac{1}{2}$ using counters and can also show this by drawing their own representations.</p>	<p>12 shared between 3 is 4.</p> <p>Also introduce division sign</p> $12 \div 3 = 4$ <p>Foundation to be shown number sentence alongside pictorial and concrete support.</p>

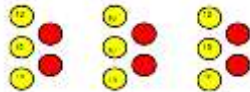


Year 2

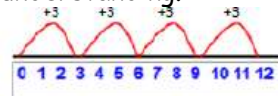
Use counters, cubes or place value counters to aide understanding.



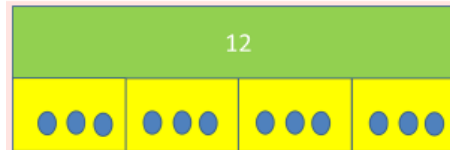
$$96 \div 3 = 32$$



Use bar modelling or number lines to support understanding.

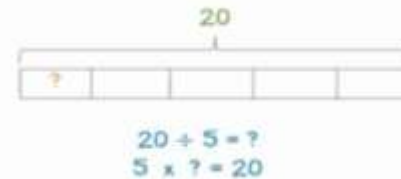


$$12 \div 3 = 4$$



$$12 \div 4 = 3$$

Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.



$$28 \div 7 = 4$$

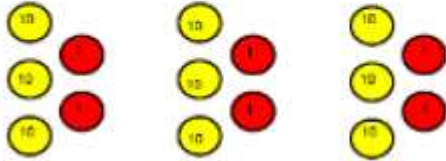
Divide 28 into 7 groups. How many groups are there?



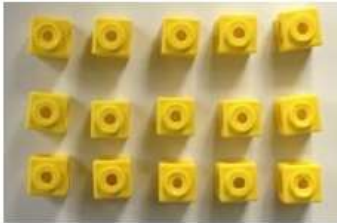
Year 3

Use cubes, counters or place value counters to aid understanding.

$$96 \div 3 = 32$$

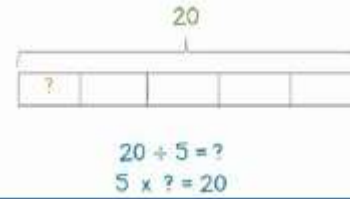


Make stronger links to division and multiplication through the use of arrays.



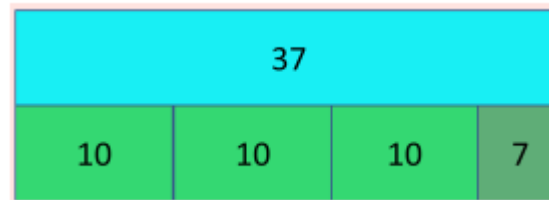
15 divide by 5 = 3

Continue to use bar modelling or number lines to support understanding.

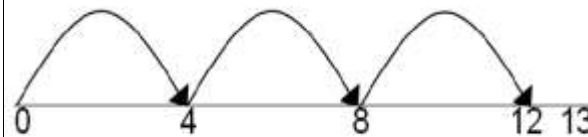


Can also be used for division with remainders

37 divided by 10



Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



Draw dots and group them to divide an amount and clearly show a remainder.



Introduce the formal layout.

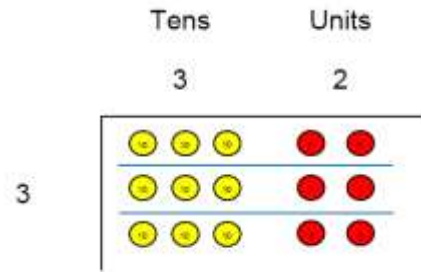
$$24 \div 3 = 8$$

8

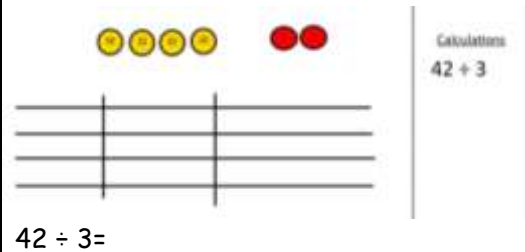
$$3 \overline{) 24}$$



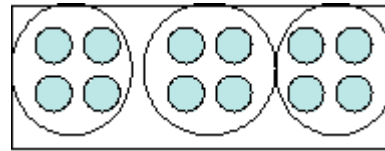
Year 4- 6



Use place value counters to divide using the bus stop method alongside



Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r}
 218 \\
 3 \overline{) 872} \\
 \underline{6} \\
 27 \\
 \underline{27} \\
 0
 \end{array}$$

Move onto divisions with a remainder.

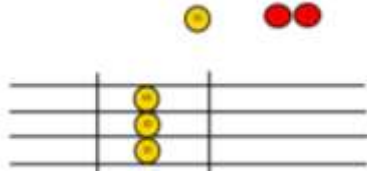
$$\begin{array}{r}
 86 \text{ r } 2 \\
 5 \overline{) 432} \\
 \underline{25} \\
 18 \\
 \underline{15} \\
 32 \\
 \underline{30} \\
 2
 \end{array}$$

Finally move into decimal places to divide the total accurately.

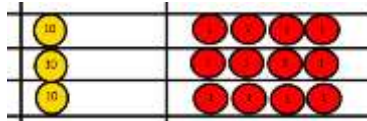
$$\begin{array}{r}
 14.6 \\
 35 \overline{) 511.0} \\
 \underline{35} \\
 16 \\
 \underline{14} \\
 21 \\
 \underline{21} \\
 0
 \end{array}$$



Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.



Year 6	Long division		
	1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
	$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \end{array}$	$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \\ -2 \\ \hline 0 \end{array}$	$\begin{array}{r} \text{h t o} \\ 18 \\ 2 \overline{) 278} \\ -2 \downarrow \\ \hline 07 \end{array}$
	Two goes into 2 one time, or 2 hundreds $\div 2 = 1$ hundred.	Multiply $1 \times 2 = 2$, write that 2 under the two, and subtract to find the remainder of zero.	Next, drop down the 7 of the tens next to the zero.
Divide.	Multiply & subtract.	Drop down the next digit.	
$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \end{array}$	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 1 \end{array}$	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$	
Divide 2 into 7. Place 3 into the quotient.	Multiply $3 \times 2 = 6$, write that 6 under the 7, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the 1 leftover ten.	



	1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.	
	$ \begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \end{array} $ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$ \begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \\ \underline{-18} \\ 0 \end{array} $ <p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract to find the remainder of zero.</p>	$ \begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \\ \underline{-18} \\ 0 \end{array} $ <p>There are no more digits to drop down. The quotient is 139.</p>	